Regularization of Neural Networks using DropConnect

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Introduction

We introduce DropConnect, a generalization of Hinton’s Dropout for regularizing large fully-connected layers within neural networks. When training with Dropout, a randomly selected subset of activations are set to zero within each layer. DropConnect instead sets a randomly selected subset of weights within the network to zero. Each unit thus receives input from a random subset of units in the previous layer. We derive a bound on the generalization performance of both Dropout and DropConnect.

Motivation

Training Network with Dropout:
Each element of a layer’s output is kept with probability $p$, otherwise being set to 0 with probability $1 - p$. If we further assume neural activation function with $a(0) = 0$, such as tanh and relu ($\ast$ is element-wise multiplication):

$$r = m \ast a(Wv) = a(m \ast Wv)$$

Training Network with DropConnect:
Generalization of Dropout in which each connection, rather than each output unit, can be dropped with probability $1 - p$:

$$r = a((M \ast W) v)$$

where $M$ is weight mask, $W$ is fully-connected layer weights and $v$ is fully-connected layer inputs.

Mixture Model Interpretation

DropConnect Network is a mixture model of $2^{|M|}$ neural network classifiers $f(x; \theta, M)$:

$$o = E_M [f(x; \theta, M)] = \sum_M p(M) f(x; \theta, M)$$

https://cs.nyu.edu/~wanli/dopc/
It is not hard to show stochastic gradient descent with random mask $M$ for each data improves the lower bound of mixture model

### Inference

**Dropout Network Inference (mean-inference):**

\[
\mathbb{E}_M[\alpha(M \ast W)v] \approx \alpha(\mathbb{E}_M[(M \ast W)v]) = \alpha(pWv)
\]

**DropConnect Network Inference (sampling):**

\[
\mathbb{E}_M[\alpha(M \ast W)v] \approx \mathbb{E}_u[\alpha(u)] \text{ where } u \sim \mathcal{N}(pWv, p(1-p)(W \ast W)(v \ast v)) \text{ i.e. each neuron activation are approximated by a Gaussian distribution via moment matching.}
\]

### Experiment Results

Experiment with MNIST dataset using 2-layer fully connected neural network:

(a) Prevent overfitting as the size of connected layers increase

(b) Varying the drop-rate in a 400-400 network

(c) Conv

Evaluate DropConnect mode! for regularizing deep neural network of various popular image classification datasets:

| Image Classification Error(%) of DropConnect v.s. Dropout |
|-----------------|------------|-----------------|
| Dataset         | DropConnect| Dropout | Previous best result(2013) |
| MNIST           | **0.21**   | 0.27     | 0.23                        |
| CIFAR-10        | **9.32**   | 9.83     | 9.55                        |
| SVHN            | **1.94**   | 1.96     | 2.80                        |
| NORB-full-2fold | 3.23       | **3.03** | 3.36                        |

### Implementation Details

Performance comparison between different implementation of DropConnect layer on NVidia GTX 580 GPU relative to 2.67GHz Intel Xeon (compiled with -O3 flag). Input and output dimension is 1024 and mini-batch size is 128. You might not get exactly the same number with my code on your machine:

https://cs.nyu.edu/~wanli/dopc/
## Efficient Implementation of DropConnect

<table>
<thead>
<tr>
<th>Implementation</th>
<th>Mask Weight</th>
<th>Total Time (ms)</th>
<th>Speedup</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPU</td>
<td>float</td>
<td>3401.6</td>
<td>1.0 X</td>
</tr>
<tr>
<td>CPU</td>
<td>bit</td>
<td>1831.1</td>
<td>1.9 X</td>
</tr>
<tr>
<td>GPU</td>
<td>float (global memory)</td>
<td>35.0</td>
<td>97.2 X</td>
</tr>
<tr>
<td>GPU</td>
<td>float (tex1D memory)</td>
<td>27.2</td>
<td>126.0 X</td>
</tr>
<tr>
<td>GPU</td>
<td>bit (tex2D memory)</td>
<td>8.2</td>
<td>414.8 X</td>
</tr>
</tbody>
</table>

Total Time includes: fprop, bprop and update for each mini-batch

Thus, efficient implementation: 1) encode connection information in bits 2) align 2D memory bind to 2D texture for fast query connection status. Texture memory cache hit rate of our implementation is close to 90%.

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### Why DropConnect Regularize Network

Rademacher Complexity of Model: \[ \max |W_i| \leq B, \max |W| \leq B, k \] is the number of classes, \( \hat{R}_t(G) \) is the Rademacher complexity of the feature extractor, \( n \) and \( d \) are the dimensionality of the input and output of the DropConnect layer respectively:

\[
\hat{R}_t(\mathcal{F}) \leq p (2 \sqrt{k} d B, n \sqrt{d} B k) \hat{R}_t(G)
\]

**Special Cases of \( p \):**

1. \( p = 0 \): the model complexity is zero, since the input has no influence on the output.
2. \( p = 1 \): it returns to the complexity of a standard model.
3. \( p = 1/2 \): all sub-models have equal preference.

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### Reference

Regularization of Neural Network using DropConnect Li Wan, Matthew Zeiler, Sixin Zhang, Yann LeCun, Rob Fergus  
*International Conference on Machine Learning 2013* [10 pages PDF] Supplementary Material Slides

CUDA code (code Sep-20-2013 update changelog)

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### Reproduce Experiment Results

The full project code is [here](https://cs.nyu.edu/~wanli/dopc/) in case you want to repeat some of the experiments in our paper. Please refer to [here](https://cs.nyu.edu/~wanli/dopc/) for how to compile the code. Some examples to run the code is [here](https://cs.nyu.edu/~wanli/dopc/). Unfortunately, the code is a little bit unorganized and I might clean up in the future. Important trained models and config files are also available [here](https://cs.nyu.edu/~wanli/dopc/) (Updated Dec-16-2013).

Zygmunt from [FastML](https://cs.nyu.edu/~wanli/dopc/) has successfully reproduce experiment result on CIFAR-10 on [Kaggle CIFAR-10 leaderboard](https://cs.nyu.edu/~wanli/dopc/) in his artical [Regularizing neural networks with dropout and with DropConnect](https://cs.nyu.edu/~wanli/dopc/).

A summary of question and my answer for hacking my uncleaned code is [Here](https://cs.nyu.edu/~wanli/dopc/)