It from Qubit:
Simons Collaboration on Quantum Fields, Gravity, and Information

1 Introduction

> *"It from bit" symbolizes the idea that every item of the physical world has at bottom — a very deep bottom, in most instances — an immaterial source and explanation; that which we call reality arises in the last analysis from the posing of yes-or-no questions and the registering of equipment-evoked responses; in short, that all things physical are information-theoretic in origin and that this is a participatory universe.*
> – John A. Wheeler, 1990 [1]

When Shannon formulated his groundbreaking theory of information in 1948, he did not know what to call its central quantity, a measure of uncertainty. It was von Neumann who recognized Shannon's formula from statistical physics and suggested the name *entropy*. This was but the first in a series of remarkable connections between physics and information theory. Later, tantalizing hints from the study of quantum fields and gravity, such as the Bekenstein-Hawking formula for the entropy of a black hole, inspired Wheeler's famous 1990 exhortation to derive "it from bit," a three-syllable manifesto asserting that, to properly unify the geometry of general relativity with the indeterminacy of quantum mechanics, it would be necessary to inject fundamentally new ideas from information theory. Wheeler's vision was sound, but it came twenty-five years early. Only now is it coming to fruition, with the twist that classical bits have given way to the *qubits* of quantum information theory.

Spurred initially by the technological promise of quantum computers, quantum information science has flourished over the past decade, yielding a range of powerful tools for analyzing and designing complex quantum systems. Its concepts and techniques have been applied with increasing frequency and sophistication in the study of quantum field theory and quantum gravity, helping to solve and sharpen old questions while raising new ones. The tantalizing hints of previous decades have given way to clear and compelling evidence that information theory provides a powerful way to structure our thinking about fundamental physics. In turn, fundamental physics provides new classes of fascinating questions for quantum information theorists to address.

Central to this dialogue is the phenomenon of quantum entanglement, the remarkable correlation possible in quantum mechanics that Einstein famously called "spooky action at a distance." The information stored in a highly entangled quantum system cannot be accessed by interacting just with its individual parts because that information is not stored in the individual parts at all, but only in the correlations among the parts. Such states generally have no succinct description; they can be exponentially complex. We have begun to understand that these features underlie the way space itself emerges from underlying microscopic building blocks. Disentangling the quantum fluctuations of two contiguous regions of space is tantamount to disconnecting the regions altogether, while entangling distant regions leads to the existence of spatial connections between them. So, in addition to the complex and highly entangled states engineered to exist inside

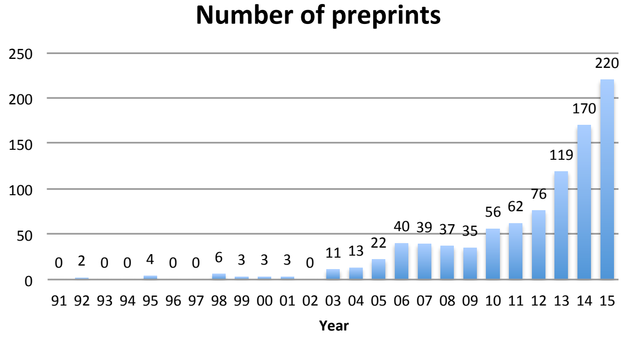

Figure 1: hep-th arXiv preprints with "entanglement" in the title. (The last entry is an extrapolation from data up to 15 March 2015.)

hypothetical quantum computers, there is another, entirely natural arena in which they play a crucial role: the physics of black holes, horizons and emergent spacetime.

Black holes and quantum gravity are remote phenomena that cannot at present be accessed experimentally, but — in an example of what might be called *the unreasonable connectivity of physics* — quantum field theory, condensed-matter physics, and quantum gravity are now all using the same tools of quantum information theory, and confronting many of the same underlying problems. This circumstance has led to a highly productive multidirectional flow of ideas among research areas. A key example is provided by the so-called holographic dualities connecting quantum gravity and lower-dimensional gauge theories. In addition to illuminating the nature of quantum gravity, these dualities have been an extremely productive tool for discovering new phenomena in quantum field theory.

Concepts from quantum information theory have driven other important advances in quantum field theory, involving a rich interplay between quantum mechanics and special relativity. This has led to profound insights into the structure of the space of quantum field theories, in particular how different theories are related under changes of scale. This question, the structure of the renormalization group, has played a central role in quantum field theory for several decades, yet in many cases important progress has come only recently, with the injection of ideas from quantum information theory.

Meanwhile, there are indications that quantum computational complexity may play a central role in the evolution of the geometry behind black-hole horizons. These developments are closely related to quantum chaos and have opened up a new way to define this elusive concept. They are also related to deep mathematical questions of complexity theory of interest in computer science. The connection of quantum gravity with complexity opens the possibility of a novel and fruitful collaboration between quantum gravity and theoretical computer science.

A rapid change is taking place in our thinking about fundamental physics that amounts to a nascent paradigm shift. The language itself is changing: fifteen years ago very few high-energy theorists used the term *qubit*, and most thought that quantum entanglement was an esoteric subject most suitable for philosophical debate. No longer. Figure 1 is a histogram showing the number of papers from the high-energy theory arXiv containing the term "entanglement" in the title. Between the year 2000 and today the growth has been exponential and far faster than the growth of the total number of papers on the arXiv. Almost all of this growth represents the connection between quantum gravity, quantum field theory, and quantum information theory.

In spite of these exciting developments, fundamental physics and quantum information theory remain distinct disciplines and communities, separated by significant barriers to communication and collaboration. These barriers are of both a historical and an institutional nature, and are exacerbated by the pigeonholed nature of governmental funding mechanisms. Given the scope of the topic, achieving the necessary systemic change to fulfill its scientific promise will require a large-scale and concerted effort, involving many of the key players on both sides.

We are proposing just such an effort. *It from Qubit: Simons Collaboration on Quantum Fields, Gravity, and Information* will be a global, intensive program bringing together some of the very best people of all generations in fundamental physics and quantum information theory. It will spur communication and education between the two communities, foster deep and sustained collaborations among their members, and nurture a new generation of scientists who will think in a new way. The result will be the creation of a new scientific discipline, leading to paradigm-changing discoveries on some of the deepest questions in physics.

2 Principal investigators

The Collaboration will attack some of the deepest questions in physics by bringing to bear the mindset and tools of quantum information theory. The members of the Collaboration include leaders in essentially all of the recent major advances at the interface of quantum information and fundamental physics. Every member of the Collaboration has a demonstrated ability and enthusiasm for reaching across domain boundaries, but each has also been carefully selected based on her or his deep and creative contributions to their own discipline. Collectively, the Collaboration has tremendous strength in quantum field theory, string theory, general relativity, quantum information theory, quantum computing, and quantum complexity theory. Our team of founding principal investigators consists of:

- **Scott Aaronson (MIT):** Aaronson is a quantum complexity theorist who has systematically explored the power of quantum computation and its variants. Particularly relevant are his studies of computation with post-selection and his demonstration that closed timelike curves make space and time equivalent for the purposes of computation.
- **Dorit Aharonov (Hebrew University):** Ever since showing that fault-tolerant quantum computation is possible in principle, Aharonov has consistently introduced new physical ideas into the field of quantum computation. Her results include a proof of the entanglement area law starting from minimal assumptions, quantum algorithms for topological invariants, and the proof of the equivalence of the adiabatic and circuit models of quantum computing.
- **Vijay Balasubramanian (UPenn):** Balasubramanian has made crucial contributions in the development of the AdS/CFT correspondence. Recently, he has made important progress on thermalization in strongly-coupled holographic field theories and in understanding how spacetime geometry is encoded in entanglement structure.
- **Horacio Casini (Bariloche):** Casini has applied a unique blend of information theoretic ideas and operator algebraic techniques to the study of quantum field theory, leading to new renormalization-group monotones and the first real proofs of both the Bekenstein and covariant Bousso bounds on spacetime information density.
- **Patrick Hayden (Stanford):** A quantum information theorist known for his results in communication and entanglement theory who, along with **Preskill**, first identified the relevance of quantum error correction to the black-hole information problem. He has since completely characterized the possible ways that quantum information can be replicated in Minkowski spacetime. Hayden will serve as Director of the Collaboration.
- **Matthew Headrick (Brandeis):** Headrick has been a pioneer in studying entanglement entropy in the holographic context. His contributions include a proof with **Takayanagi** that the holographic formula for entanglement entropy respects strong subadditivity, and important work on generalized (Rényi) entropies in holographic theories.
- **Juan Maldacena (IAS):** In one of the most cited papers in all of physics, Maldacena proposed the AdS/CFT correspondence, providing a precise equivalence between quantum gravity in asymptotically anti-de Sitter space and a conformal field theory in a lower dimension. His recent contributions include a proof of the Ryu-**Takayanagi** entanglement-entropy formula and foundational studies of quantum chaos relevant to the dynamics of black holes.
- **Alex Maloney (McGill):** An expert in string theory and quantum gravity, Maloney is a leader in the area of lower-dimensional models of quantum gravity. His recent work includes a proposed novel measure of entanglement (charged Renyi entropies, with **Myers**) and a proof with **Hayden** and **Headrick** that field theory states encoding classical geometries obey constraints on their entanglement known as the negativity of tripartite information.
- **Don Marolf (UCSB):** Marolf brings a unique mix of expertise in classical gravity, quantum gravity, and string theory. His recent collaboration with **Polchinski** focusing on entanglement in the context of black holes led to the "firewall" paradox which suggests macroscopic quantum-mechanical effects in gravitational physics and has the potential to completely change our understanding of black holes.
- **Rob Myers (Perimeter):** Myers is an expert on gravity and string theory with a large number of influential works on D-branes, black holes, and the AdS/CFT correspondence. He has important recent contributions to understanding the connection between entanglement entropy and spacetime geometry, several in collaboration with **Van Raamsdonk**.
- **Jonathan Oppenheim (UCL):** Responsible for the operational interpretation of the fact that quantum conditional entropy can be negative, which subsequently played a key role in identifying the holographic dual of curve length. He has also generalized entanglement theory to other contexts, building a thermodynamics of nanoscale systems.

- **Joe Polchinski (KITP):** Polchinksi has made many seminal contributions to quantum field theory and string theory, including the discovery of D-branes, which led to the AdS/CFT correspondence. With **Marolf**, he recently formulated the "firewall" paradox, which has inspired much recent work at the intersection of quantum information and quantum gravity.
- **John Preskill (Caltech):** Originally a particle theorist, Preskill is one of the leading figures in quantum information science. His early and ongoing contributions to the development of fault-tolerant quantum computation have been crucial. In addition, he has used information-theoretic ideas to characterize the physically realizable observables in quantum field theory and designed the first quantum algorithms for calculating scattering amplitudes.
- **Lenny Susskind (Stanford):** One of the founders of string theory, Susskind was responsible, along with 't Hooft, for formulating the holographic principle ultimately realized in the AdS/CFT correspondence. He also developed many of the techniques used for lattice simulations of quantum field theory. Most recently, he has been actively exploring the relevance of tensor networks and computational complexity to the structure of spacetime.
- **Tadashi Takayanagi (Kyoto):** Takayanagi first conjectured the fundamental relationship between areas of bulk minimal surfaces and boundary entanglement entropy in holographic spacetimes, thereby systematically linking bulk geometry and boundary information and providing the foundation for much of this proposal.
- **Mark Van Raamsdonk (UBC):** In an influential essay, Van Raamsdonk first articulated the idea that space is stitched together by entanglement, and he has since developed increasingly sophisticated evidence for the proposal. He has recently shown that Einstein's equations follow from the properties of boundary entanglement in holographic theories together with the holographic formula for entanglement entropy.

In the following section, the PIs' individual contributions are put into a larger context. We have also budgeted to allow one or two new PIs to join the Collaboration during the course of the program. Our intention is that these will be newly hired junior faculty likely drawn from the current cohort of postdoctoral fellows, some of whom have already distinguished themselves as leaders in this field.

3 Scientific context and objectives

Here we will explain five motivating deep questions that will be addressed by the Collaboration, including a brief summary of important recent progress. While we will sketch some research directions, detailed descriptions of the Collaboration's immediate research objectives can be found in section 4.

3.1 Does spacetime emerge from entanglement?

One of the greatest challenges in theoretical physics is the quantum-mechanical description of gravity. Truly remarkable progress on this problem has come from the matrix-model formulations of string/M-theory, invented by **Susskind** and collaborators [2] and holographic (or AdS/CFT) dualities, discovered by **Maldacena** [3]. In these, the first non-perturbative formulations of quantum gravity, spacetime is not built in from the start, but rather *emerges* from the strongly-coupled quantum dynamics of ordinary non-gravitational systems. Recent work has revealed a stunning result: the structure and geometry of the emergent spacetime is related directly and quantitatively to the entanglement among the fundamental degrees of freedom of the theory. This implies that classical spacetime does not exist without quantum entanglement! Even the Einstein equation — the fundamental law of classical gravitation — can be understood directly from the physics of quantum entanglement.

This profound insight implies that two previously disconnected subjects — quantum gravity and quantum information theory — are in fact different aspects of the same physics. Various members of the collaboration have played a central role in these developments:

- **Maldacena**'s suggestion that black holes with two asymptotic regions are described by special entangled states of two non-interacting quantum systems [4] provided an early connection between entanglement and spacetime.

- The holographic entanglement entropy formula proposed by Ryu and **Takayanagi** [5] provided a direct quantitative relation between entanglement entropy — a measure of quantum entanglement — and spacetime geometry. This formula has been tested, applied, extended, and derived by **Balasubramanian, Casini, Hayden, Headrick, Maldacena, Maloney, Marolf, Myers, Takayanagi**, and others ([6, 7, 8], among many other works).

- Inspired by the Ryu-Takayanagi formula, **Van Raamsdonk** proposed that spacetime geometry emerges from entanglement [9]. **Maldacena** and **Susskind** extended this idea in the "ER = EPR" conjecture relating wormholes, also known as Einstein-Rosen bridges, to entanglement in the form of Einstein-Podolsky-Rosen states [10].
- **Headrick**, **Takayanagi**, and others [11] understood that sophisticated properties of quantum information, such as strong subadditivity, are encoded in properties of spacetime geometry. They, along with **Van Raamsdonk** and collaborators, have understood that certain energy conditions in gravity are connected with strong subadditivity of entanglement entropy and positivity/monotonicity of relative entropy [12, 13, 14].
- **Balasubramanian, Hayden, Headrick, Myers,** and collaborators have built on the Ryu-Takayanagi formula to understand precisely how gravitational spacetimes can be reconstructed directly from information-theoretic properties of dual field theories [15, 16].
- **Myers**, **Van Raamsdonk**, and collaborators have shown [17, 18] that the Einstein equation can be derived from certain constraints on entanglement entropy uncovered earlier by **Casini**, **Myers** and collaborators [19].
- Almheri, Dong, and Harlow [20] have suggested that the boundary state in AdS/CFT is a form of quantum error-correcting code. **Preskill** [21] and **Polchinski** [22] have begun working to understand this more precisely and to explicitly construct examples of states with the required properties.

Numerous fundamental questions are now ripe for exploration, including:

- Which states of ordinary quantum systems consistently encode a dual spacetime?
- For quantum states which do admit a dual spacetime, how is information about spacetime encoded?
- What are the implications of basic constraints on entanglement structure, such as strong subadditivity and positivity/monotonicity of relative entropy, on gravitational physics?
- Finally, it is likely that the lessons learned in understanding the connections between gravity and entanglement will give us insights into what kinds of quantum systems and entanglement structures might represent cosmological space-times. This may lead to progress on some of the deepest puzzles in physics, such as the cosmological constant problem, the nature of the big bang, and the origin of structure in the universe.

3.2 Do black holes have interiors? Does the universe exist outside our horizon?

Black holes have been a crucial arena for the development of quantum gravity. The need to understand the microscopic nature of black-hole entropy and to resolve the information loss paradox have driven research in this field for the past four decades, and have led to the discovery of deep and unexpected principles. Indeed, the holographic principle described above was originally motivated, in work by 't Hooft and **Susskind**, by considerations of black-hole physics [23, 24]. A more detailed study of the relations between black holes and D-branes led **Maldacena** to the discovery of AdS/CFT duality [3].

The black hole information paradox — the need to reconcile the existence of Hawking radiation with unitary quantum mechanics — continues to be a merciless proving ground for ideas about quantum gravity. Quantum information theory has played a key role in sharpening this paradox and exploring possible solutions. Page, **Preskill**, and **Susskind** realized early on that unitary black hole evaporation would create an apparent violation of the no-cloning theorem of quantum mechanics, but one that appeared to be unobservable even in principle [25]. **Hayden** and **Preskill** revisited this question from the perspective of quantum error correction, again finding the effect to be unobservable but this time only barely so [26]. In 2012, however, **Marolf**, **Polchinski**, and their students (AMPS) considered a slightly different thought experiment and unleashed a crisis in theoretical physics [27]. A careful study of entanglement, unitarity, and locality seems to predict that the black-hole horizon must be replaced by a firewall, an abrupt end to spacetime. Their work has challenged the established consensus that the AdS/CFT duality provides a complete non-perturbative definition of quantum gravity.

As these examples show, the study of black holes and quantum gravity has increasingly relied on notions from quantum information theory:

- Harlow and **Hayden** argued that constraints on computational complexity can make the AMPS thought experiment impossible [28]. However, **Marolf**, **Polchinski**, and collaborators have argued that these constraints do not apply for black holes in AdS [29], and **Oppenheim** and Unruh have argued that they can be avoided by incorporating precomputation [30].

- **Preskill** and Lloyd [31] have shown that the proposal of Horowitz and **Maldacena** [32] to impose a final state boundary condition at the black hole singularity allows quantum cloning and so avoids the AMPS paradox. Issues of backwards causality, both inside the black hole and potentially in the exterior as well, remain.

- **Oppenheim** and collaborators [33, 34] and **Preskill**, have considered possible modifications of quantum mechanics, including the possibility that information loss can actually occur in a consistent way.

- **Susskind** argued that entanglement is not enough to characterize the evolving and growing geometry behind the black hole horizon. The other essential feature of quantum mechanics — the capacity for exponential complexity — is key [35]. Over the last year or two, evidence has emerged to suggest that there is a remarkable duality between quantum computational complexity and spatial volume.

- **Balasubramanian**, **Hayden**, **Maloney**, **Marolf**, and **Susskind** have initiated investigations into the role of multipartite entanglement in black-hole physics [36, 37].

Going forward, we expect to either answer or make substantial progress on the following questions:

- How does complexity grow in black holes and for how long a time?
- What is the connection between quantum chaos, complexity, and gravity?
- Can the firewall can be avoided, as some have argued, by representing the black hole interior using state-dependent operators rather than the linear operators of ordinary quantum theory?
- What kinds of black holes, with what structures in their interiors, can one produce as the dominant saddles of bulk Euclidean path integrals with specified boundary conditions?
- Which patterns of multipartite entanglement can be associated with smooth multi-black-hole geometries. Which patterns, for example, have smooth interiors?

While holographic dualities have been useful in thinking about these questions, they lean on the crutch of having a timelike asymptotic boundary. The same questions apply to cosmological spacetimes which have no such boundary. These issues bear directly on important questions in cosmology, such as the measure problem in eternal inflation: how does one extract useful predictions from theories in which the universe is so large that any conceivable occurrence happens somewhere? What lessons can be drawn from the finite gravitational entropy of cosmological horizons, and do they ameliorate this large-universe problem?

3.3 What is the information-theoretic structure of quantum field theories?

While the study of entanglement entropy in quantum field theories began with attempts to understand black-hole entropy [38, 39], the subject has since flourished into a major area in its own right, with ramifications from condensed-matter theory to string theory. It provides a new perspective on field theories, different from that provided by traditional quantities such as correlation functions and scattering amplitudes. Here are just two out of the many important applications so far:

- Entanglement entropies have been used by **Preskill**, **Aharonov**, and others [40] to diagnose topological order in condensed-matter systems, providing a new window onto strongly coupled systems.

- A natural application of information theory to quantum field theory is in the study of the effect of renormalization-group flows, where one seeks to characterize precisely the loss of degrees of freedom due to coarse-graining. Entanglement entropies have been used by **Casini**, **Myers**, and others to demonstrate the irreversible structure of renormalization group flows in two and three dimensions [41, 42] and to characterize the approach to thermal equilibrium.

- Relative entropy has also recently been explored, and been used by **Casini** to prove a rigorous and well-defined version of the Bekenstein bound [43].

Those advances prove the power of the information-theoretic approach, but there is a great deal more to be done.

- Can the irreversibility of renormalization-group flows in four dimensions be characterized information-theoretically?
- As in the holographic setting, the important problem of understanding patterns of multipartite entanglement has just begun to be explored [44, 45].
- Previous work has focused primarily on entanglement entropies in the ground state. We will study the problem in higher dimensions and also study the time evolution of the entropies, paying particular attention to the effects of integrability and chaos.

3.4 Can quantum computers simulate all physical phenomena?

Despite being enormously powerful, modern computers are frequently useless for simulating even moderately-sized quantum systems. This computational intractability is one of the most serious limitations of modern science. On the other hand, it seems plausible that a general-purpose quantum computer will be able to efficiently simulate any process that occurs in nature. Quantum algorithms already exist for faithfully simulating chemistry and predicting the properties of materials. However, while quantum algorithms for simulating discrete systems are well-developed, their extension to field theories and curved, or even emergent, geometry is just beginning. The Collaboration will develop quantum computing techniques for simulating gauge field theories, including the standard model of particle physics, and string theory.

Our team is well-placed to attack this problem. In addition to its strength in quantum field theory and string theory, members have played central roles in identifying and achieving the limits of quantum computation and simulation:

- **Aharonov** and others have developed algorithms for simulating topological quantum field theories [46] and non-standard models of quantum computation [47].
- **Headrick** has developed novel numerical methods for solving Einstein's equations which are the state of the art for some problems involving static black hole solutions [48].
- **Preskill** and collaborators have designed quantum algorithms for simulating scattering processes in simple interacting field theories [49].
- Anticipating that it will be necessary to formulate a theory of computational complexity adapted to the complications of curved spacetime, **Aaronson** has already precisely characterized the computational power of closed timelike curves and future boundary conditions [50, 51].

The ultimate question is whether quantum computers can accurately simulate nonperturbative phenomena in quantum gravity, where spacetime is subject to strong quantum fluctuations? If the answer is Yes, then simulating quantum gravity will be an especially worthy application for quantum computers. If the answer is No, then the computational power encoded in the laws of physics surpasses what is captured by our currently accepted quantum computational models.

3.5 How does quantum information flow in time?

While entanglement theory has proved to be a very effective tool for fundamental physics, important limitations have been revealed by applying it to physical systems very different from the communication and computing scenarios for which it was originally invented. For example, because of the theory's emphasis on states, its methods are much more powerful for analyzing correlations in space than in time. Motivated by the need to quantify correlations and physical constraints in quantum field theory and holography, we must work to produce a new, more potent form of quantum information theory. Progress in this direction has already begun:

- **Preskill** and collaborators have studied entanglement in the presence of superselection rules [52].
- **Oppenheim** has been systematically generalizing entanglement theory by abstracting away the notion of a theory that quantifies resources when different types of constraints are imposed on physical transformations. He has thereby succeeded in building usable theories of both microscopic and far-from-equilibrium thermodynamics [53].
- **Balasubramanian** and **Van Raamsdonk** have studied momentum-space entanglement in interacting field theories [54].

These and subsequent investigations have taught us that similar information-theoretic principles can be used to quantify nonlocality, asymmetry, and athermality. They also provide a blueprint for extending entanglement theory to the study of correlations in time. If entanglement does indeed hold space together, then finding its timelike analog will be necessary to explain the emergence of time.

It has recently been shown that the dynamics of black hole horizons is governed by fluid dynamics at long distances. This is only the most recent development pointing to a strong connection between thermodynamics, statistical mechanics, and gravity. The relationship, as presently understood, only holds at long distances along the horizon. However, this suggests that there should be a more general notion of coarse-graining, or approximation, which leads precisely to gravity everywhere, not just along black hole horizons. But gravity is not just thermodynamics; it goes beyond thermodynamics because it encodes entanglement, which is a purely quantum property. One possible slogan is that "gravity is the fluid dynamics of entanglement." For now, these are just words, but we aim to turn them into precise formulas.

4 Projects

Guided by the overarching scientific questions described in the previous section, the members of the Collaboration will identify specific projects that are likely to be amenable to progress in the near term. The identification of such projects will be bottom-up, dynamic, and ongoing throughout the duration of the Collaboration. Each project be led by a convener, who will be responsible for organizing monthly virtual meetings as well as occasional physical ones, and will have a dedicated wiki page for asynchronous collaboration (see subsection 5.2 on Collaboration activities below). In addition to members of the Collaboration, a project may involve selected outsiders with relevant expertise. While significant progress will, as usual, be reported publicly in the form of papers and conference presentations, interim progress will be reported internally to the Collaboration at the whole-Collaboration meetings and online via the wiki. Whereas typical scientific collaborations are private and known only to the researchers involved, these projects will be known and accessible to the whole Collaboration; any member may join any virtual or physical meeting of any project, and potentially contribute to its progress. In this way we will leverage the knowledge and skills of the entire Collaboration. Furthermore, by monitoring the research projects as they develop, we will assess the Collaboration's progress towards its larger scientific goals.

We have identified an initial set of eight well-defined research projects of broad interest to the members of the Collaboration on which work will begin immediately. These are described below, with the convener and initial participants listed; in almost every case this includes a mixture of high-energy physicists and quantum information theorists. We expect substantial progress on each of these projects within the first two years of the Collaboration.

1. **Using tensor networks to reconstruct bulk geometry from boundary states**

   Several years ago, a striking similarity was observed between the hyperbolic spatial geometry of anti de Sitter space and the structure of tensor networks being used to represent the ground states of conformal field theories in numerical investigations. These MERA (Multiscale Entanglement Renormalization Ansatz) states are designed to faithfully support the entanglement of the CFT. Given the close connection between entanglement and geometry that has emerged since, the connection is natural. Whether the striking similarity can be elevated to a mathematical equivalence, however, remains unclear. We propose to explicitly construct the MERA representation of a CFT state with a holographic dual and thereby determine how faithfully it represents the geometry.

   More specifically, we will develop a coarse-grained MERA network starting from the boundary state using the techniques of one-shot information theory, which governs compression and entanglement distillation in arbitrary quantum states. The first step of the argument will be to show that the one-shot quantities governing the operational information-theoretic quantities are well-approximated by their von Neumann counterparts. Next, we will use the probabilistic method to construct optimal disentanglers.

   Convener: Marolf. **Participants:** Aharanov, Balasubramanian, Hayden, Maloney, Myers, Polchinski, Susskind, Takayanagi

2. **AdS/CFT and quantum error correction**

   Recently, **Preskill** and collaborators discovered how to relate error-correcting codes to the holographic properties of space [21]. It has been suggested that the holographic encoding of bulk AdS space in the dual CFT has the form of a quantum error-correcting code, and tensor networks have been identified that implement such codes. This observation undoubtedly has deep implications for the holographic principle, and it may also offer new possibilities for applications to practical codes. This unexpected connection between gravity and (quantum) computer science is very new and it opens up many fundamental and practical questions. One urgent theoretical question is how bulk locality on smaller scales than the AdS radius of curvature emerges.

We will improve the construction to reflect more features of the full AdS/CFT correspondence, including extending it from the coarse network models of [21] to the continuum CFT. Additional questions to explore are the relation to gauge invariance, and the inclusion of time dependence in the boundary and the bulk. We will also study information-theoretic implications of introducing a fixed non-dynamical conical singularity into the tensor network, and in particular of the existence of a bulk region in such geometries that cannot be probed by minimal geodesics anchored at the boundary.

Convener: Polchinski. **Participants:** Balasubramanian, Casini, Hayden, Maldacena, Myers, Preskill, Susskind, Van Raamsdonk

3. **Chaos and the black hole horizon**

   It has been a long-standing question to understand how the existence of the black hole horizon is reflected in the dual CFT. Recently, Shenker, Stanford, Kitaev, and **Maldacena** have provided a sharp answer: the chaotic (Lyapunov) behavior of the CFT is a direct reflection of the horizon geometry [55]. Immediate goals are to understand the bulk interpretation of Kitaev's random spin model, and to find more conventional matrix-model examples. Of particular interest is the emergent approximate conformal symmetry in Kitaev's model that governs the near-horizon geometry of near-extremal black holes. We propose to understand better the action of this symmetry. In principle this symmetry, should allow us to go behind the horizon.

   Convener: Maldacena. **Participants:** Maloney, Oppenheim, Polchinski, Susskind

4. **Operational interpretation of entanglement entropy in gauge field theories**

   Entanglement is one of the most important tools for classifying the behavior quantum field theories at short and long distances. In the case of gauge theories it is also currently playing a major role in understanding the connectivity of space in holographic theories of quantum gravity. However, the definition of entanglement entropy has been found to be ambiguous in lattice gauge theories, depending on the details of how one assigns an algebra of observables to a spatial region. Moreover, the entanglement entropy is known to contain contributions that do not correspond to distillable entanglement because of selection rules. Algebraically, the situation is fairly well understood but the physical and information-theoretic interpretation of the mathematics remain obscure. We will address both issues: how much entanglement can be distilled, and what is the interpretation of the rest of the entropy?

   Convener: Casini. **Participants:** Hayden, Headrick, Oppenheim, Preskill

5. **Constraints on gravitational spacetimes from entanglement inequalities**

   In the AdS/CFT correspondence, the geometry of spacetime is related to the entanglement structure in the dual field theory via the Ryu-**Takayanagi** formula. The entanglement structure obeys fundamental constraints, such as the strong subadditivity of entanglement entropy, and the positivity and monotonicity of relative entropy. Using the correspondence, these should translate to fundamental constraints on the geometry of spacetime. Preliminary evidence suggests that these take the form of certain energy conditions. We will study these constraints as completely as possible. Moreover, similar relations are required by the hypothesis that QFTs should satisfy a quantum focussing condition. Their bulk implications will be studied as well, with holography being used as a tool to support or falsify this hypothesis.

   Convener: Van Raamsdonk. **Participants:** Maloney, Marolf, Myers, Takayanagi

6. **Building models of theories with information destruction**

   Virtually all proposals to solve the black hole information problem assume that information cannot be destroyed. Very little consideration has been given to the alternative, although the AMPS result might be pushing us in this direction. Much of the reason that so little work has been done in this area is the existence of known obstructions to fundamental theories of information destruction — it was believed that they either violate conservation laws or are non-local. Some ways around these obstructions have been proposed by Unruh and Wald and by Reznik and **Oppenheim**; however, one likely has to consider non-Markovian theories, and more sophisticated ones. We will explore how energy can naturally be conserved in a non-Markovian theory while still destroying sufficient amounts of information. We will also study what aspects of the AdS/CFT conjecture continue to hold in such theories.

Convener: Oppenheim. **Participants:** Marolf, Polchinski, Preskill

7. **Multipartite entanglement in quantum field theory and gravity**

   Holographic dualities suggest that Einstein-Rosen bridges are manifestations of certain kinds of entanglement. However, this raises questions of consistency such as what happens to an Einstein-Rosen bridge when a measurement is performed at one end, thus breaking the entanglement. The answer involves multipartite, GHZ-type, entanglement rather than just entanglement between pairs of systems. Multipartite entanglement is much more complicated and poorly understood than the bipartite case. The connection between entanglement and geometry therefore offers a potentially powerful tool for making progress on a longstanding problem in quantum information theory. In holographic settings, there are indications that multipartite entanglement is intrinsically associated with matrix degrees of freedom and may be related to the detailed dynamics of the theory. The corresponding multipartite entanglement structures will be explored in tractable model CFTs that include free theories, the Ising model, and perhaps symmetric orbifolds.

   Convener: Maloney. **Participants:** Balasubramanian, Headrick, Marolf, Susskind

8. **Circuit complexity and the validity of general relativity at late times**

   Scrambling is the short-time manifestation of quantum chaos. The long-time manifestation is the growth of complexity. In black holes, it has been suggested that this growth is related to the growth of the interior of Einstein-Rosen bridges. The growth is ultimately bounded exponentially by the finite size of the system, but no one knows how long the complexity actually increases and whether the bound is saturated. The answer to this question governs the limits of classical geometry over very long time scales. Recently, **Aaronson** and **Susskind** have showed that the complexity really does become superpolynomial, assuming (1) the unitary implements a computationally-universal cellular automaton, and (2) the complexity class PSPACE is not contained in PP/poly. This work opens up a striking connection between computational complexity and quantum gravity, and suggests numerous open problems. For example, what can we say about the complexity of *approximately* preparing the state? Also, is there a believable complexity assumption that implies that the complexity really increases *linearly* with time?

   Convener: Susskind. **Participants:** Aaronson, Balasubramanian, Hayden, Maloney, Marolf

5 Organization and activities

5.1 Administrative structure

The Collaboration will be overseen by a Director, a Deputy Director, and an Executive Committee of five members. They will set the overall scientific direction of the Collaboration as the research proceeds, and, in consultation with the general membership, they will make important decisions, such as: reallocating funding; changing the postdoc hiring process; changing the organization of the research projects; adding a new PI or removing one whose scientific work no longer contributes to the goals of the Collaboration; and changing the Director or Deputy Director or members of the Executive Committee. The Director and Deputy Director will manage the administration on a day-to-day level.

Initially, the Director will be Patrick Hayden, the Deputy Director will be Matthew Headrick, and the Executive Committee will be composed of Dorit Aharonov, Juan Maldacena, Rob Myers, Joe Polchinski, and John Preskill. There will also be other committees as needed, such as a postdoc selection committee and organizing committees for the schools/workshops, whose membership will rotate among the PIs.

5.2 Activities

The goal of the Collaboration is to make progress on a set of scientific questions at the interface between quantum information theory and fundamental physics. There does not yet exist an established scientific discipline in this area, and there are very few scientists who are expert in both fields. Therefore, our task is not as simple as assembling a set of experts in a given field. Instead, what we are trying to do is novel and rather ambitious: to intentionally create a new scientific discipline. There is no roadmap for doing this. We have a general strategy and a plan for carrying it out, but as we go along we are bound to learn lessons about how best to achieve our goals, and to adjust course accordingly.

Each of the activities below is specifically designed to help realize one or more of the following goals:

1. to bring together the PIs, postdocs, and students to learn from each other and make progress on the scientific questions described in the proposal;
2. to train a new generation of scientists conversant in both quantum information theory and fundamental physics;
3. to enable research on the questions by supporting the PIs, postdocs, and students.

- **Meetings.**
  - The Collaboration will install teleconferencing equipment at each PI's institution (where it is lacking), to enable productive virtual meetings and seminars, and to enable members to participate in physical meetings when they cannot attend in person.
  - The PIs and postdocs will attend a three day kick-off meeting in the fall of 2015 at Stanford, will serve both scientific and organizational purposes. The breadth of the Collaboration's membership is such that some of our PIs have yet to meet in person. The meeting will serve to inform all participants of the current state-of-the-art in their own fields of expertise and dive into the process of making progress on the problems articulated in this proposal.
  - The PIs will attend an annual two-day meeting at the Simons Foundation headquarters, both for organizational purposes and to report our scientific work to the Foundation. The convener of each project will report to the rest of the Collaboration and to the Foundation on the status of his/her project.
  - We will hold three two-week whole-Collaboration workshop/schools: at the Perimeter Institute in summer 2016; in Bariloche, Argentina in January 2018; and at Kyoto University in summer 2019. Each meeting will combine a school with a workshop. The structure will consist of lectures in the morning, followed by tutorials for the students and workshop seminars in parallel in the afternoon. We expect all or almost all of the PIs, their graduate students, and Collaboration postdocs to attend these meetings. There will be 40–50 students attending the school. The lectures will be given primarily by Collaboration PIs, although we may invite outside speakers as well.

    The schools will play the dual role of providing the PIs (and postdocs) with an education in the fundamentals of the fields complementary to their own background, and of educating the younger generation in both fields. Furthermore, by attending the workshop, the more advanced students attending the school will have the opportunity to be exposed to cutting-edge research. We anticipate that the subject matter of the schools will vary from year to year and that the final school will be somewhat more advanced than the previous two. This will enhance their educational utility and ensure that students can productively return year after year.
  - The convener of each project will organize a monthly virtual meeting and occasional (roughly annual) physical ones. For example, there will be a meeting in Montreal in September 2015 on project 7; in Santa Barbara in January 2016 on projects 1 and 2; in Kyoto in June 2016 on project 5; and at Stanford in January 2017 on projects 3 and 8.
  - Each year, the postdocs (see below) will hold an informal gathering.

In summary, the minimal schedule of physical meetings for the first 18 months of the Collaboration is as follows (project meetings are denoted PM):

Dates	Event	Location
14–17 Sept 2015	PM: Multipartite entanglement and geometry	CRM, Montreal
12–14 Oct 2015	Collaboration kick-off meeting	Stanford
4–7 Jan 2016	PM: Tensor networks and quantum error correction in holography	KITP, Santa Barbara
Spring 2016	Postdoc gathering	TBD (postdoc-organized)
June 2016	PM: Constraints on spacetimes from information theory	Kyoto
Summer 2016	School and workshop	Perimeter Institute, Waterloo
Fall 2016	Full Collaboration meeting	Simons Foundation
Jan 2017	PM: Chaos, complexity, and general relativity	Stanford

There will likely be other project meetings as well (such as on projects 4 and 6), although these have not been scheduled yet. It is worth noting that, as detailed in their biosketches, the PIs already have significant experience organizing conferences, workshops, and schools that bring together high-energy physicists and quantum information theorists; in fact, almost all meetings to date in this nascent area have been organized by one or more of its members.

• **Postdocs.** The Collaboration will establish a prestigious postdoctoral program, called the *Simons Qubit Fellowship*, with the explicit purpose of fostering intellectual movement between quantum information theory and fundamental physics. The hiring process will be coordinated through a Collaboration committee, but each postdoc will be hired, employed, and mentored by a particular PI. In addition to attending the whole-Collaboration meetings, each Fellow will be funded (and required) to spend one month of each year visiting other PIs of the Collaboration. As mentioned above, in order to foster research links among the postdocs, they will also have their own annual informal workshop, at which the emphasis will be to work on joint projects rather than formally presenting established results.

Furthermore, at the beginning of the Collaboration, a select group of postdocs, who are currently working with one of the PIs and whose work matches the scientific goals of the Collaboration, will be promoted to being Simons Qubit Fellows. This will help us to start building the intellectual community of the Collaboration on Day One. (The initial class of fellows will receive at most 50% of their funding from the Collaboration.) The proposed inaugural class of postdocs includes two speakers from the recent Simons Symposium on Quantum Entanglement in Puerto Rico, who exemplify the quality and spirit of the fellows we aim to recruit: Brian Swingle and Beni Yoshida. A condensed-matter theorist by training, Swingle first proposed the geometrical holographic interpretation of scale-invariant tensor networks as a PhD student. Yoshida is a leading expert on the construction of exotic quantum error-correcting codes with extremal properties, who has just recently begun applying those skills to models of the AdS/CFT correspondence. The rest of the inaugural class is equally impressive: Henry Maxfield has been making significant progress on determining the correct generalization of the Ryu-Takayanagi formula to time-dependent spacetimes, Fernando Pastawski worked with Yoshida to discover the AdS/CFT code, and Douglas Stanford has played the leading role in establishing the information theory-inspired fast-scrambling conjecture connecting quantum information and quantum chaos.

• **Long-term visits.** One of the best ways to foster communication and collaboration is for scientists to make long-term visits to each others' research groups. In order to encourage such visits, the Collaboration will cover the costs associated with any visit of two weeks or longer by any PI, postdoc, or student to the research group of another PI. Furthermore, since family constraints are often significant barriers to such visits, the Collaboration will (on the model of the Simons Fellows in Theoretical Physics program) contribute toward the travel, housing, and childcare costs of family members during such long-term visits. Likewise, on the model of the KITP Graduate Fellowship, the Collaboration will fund semester-long visits (including a stipend supplement for living expenses and travel expenses) by the PIs' graduate students to the research group of another PI. Each PI will be afforded one such fellowship over the course of the Collaboration.

• **Web activities.** The Collaboration will maintain a private wiki, which will be accessible to all members of the Collaboration. The wiki will serve two purposes. First, there will be a space for each project, to facilitate efficient asynchronous collaboration, while allowing any interested member to follow (and perhaps participate in) the conversation. (Non-members who are involved in a particular project will also have access to the relevant wiki page.) Second, since one of the goals of the Collaboration is for us to learn from each other, especially across disciplines, there will be a space for members to ask and answer questions. In order to encourage people to ask "stupid questions", it will not be publicly accessible. However, an edited version ("best of") will be made public as a resource for researchers outside the Collaboration. This will be hosted on the Collaboration's website, which will also contain explanations of the work of the Collaboration suitable for the general public. The content on the website will be maintained by the Collaboration's administrative assistant, along with student and postdoc members of the Collaboration. The Collaboration will also create a YouTube channel, where all school lectures, workshop talks, and real and virtual seminars will be publicly available.

5.3 Budget

The Collaboration requests a total of \$10 million over four years. Each PI is allocated \$105,000 per year to support their research toward the scientific goals of this Collaboration, including graduate students, the Simons Qubit Fellows described above, and up to one month of summer salary. (The director and deputy director receive an additional \$15,000 per year.) This includes funds reserved at the Simons Foundation to add a new PI in year 2 and another in year 3.

The remaining funds are used for meetings, long-term visits, virtual meeting technology, and a quarter-time administrative assistant. These are distributed as follows: \$5,000 per year directly to each PI; \$262,000 to Rob Myers for the first school/workshop to be held at Perimeter Institute; \$677,000 to the Director, Patrick Hayden, to be adminstered by Stanford; and \$1,351,000 to be held at the Foundation and disbursed to organizers of future meetins and schools. The use of these funds is described in more detail in the Director's Budget Justification. Overall, over 25% of the budget is devoted to meetings, school, and visits within the Collaboration.

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